Exact bosonic Matrix Product States (and holography)

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Outline

What are Matrix Product States?

Motivation

understanding holography?

some questions for integrable QFT's

General construction of exact bosonic MPS

The Klein-Gordon harmonic chain

Exact MPS and the transfer matrix

The correlation function $\langle \phi_0 \phi_m \rangle$

Comments on holographic interpretation

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What are Matrix Product States?

- Consider a 1D spin chain system of length L (L is large, perhaps infinite) with some hamiltonian. One is interested in finding the ground state wavefunction
- The wave function $|\Psi\rangle = \Psi_{i_1 i_2 \dots i_L} |i_1 i_2 \dots i_L\rangle$ has exponentially many components. These components can be understood as defining a rank L tensor, which can be pictorially represented as

$\Psi_{\dots i_1 i_2 \dots i_5 \dots} =$

- Tensor networks provide variational ansatzae with less components e.g. Matrix Product State (MPS) is of the form

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- As we increase the bond dimension D ($\chi_i = 1 \dots D$), we get better and better description of the ground state

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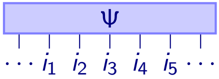

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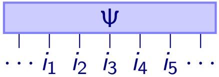
$\Psi_{\dots i_1 i_2 \dots i_5 \dots} =$ 

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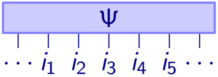
$\Psi_{\ldots i_1 i_2 \ldots i_5 \ldots} =$ 

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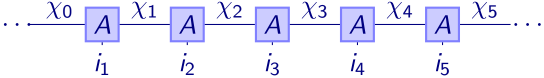
$\Psi_{\ldots i_1 i_2 \ldots i_5 \ldots} =$ 

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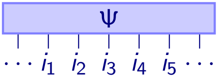
$\Psi_{\dots i_1 i_2 \dots i_5 \dots} =$ 

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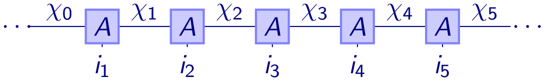
$\Psi_{\dots i_1 i_2 \dots i_5 \dots} =$ 

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Motivation — understanding holography?

- Tensor network constructions have been proposed as a way to microscopically understand holography...
- Swingle proposed a compelling link between holography and MERA
- Nozaki, Ryu, Takayanagi defined an underlying holographic metric in terms of cMERA
- ...
- Pastawski, Yoshida, Harlow, Preskill proposed isometric quantum codes

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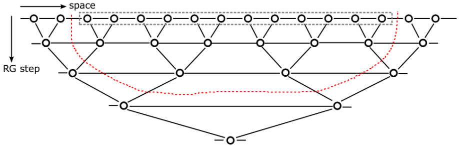

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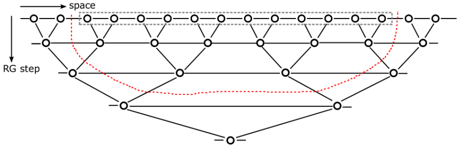

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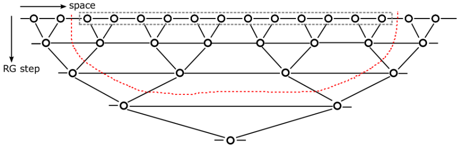

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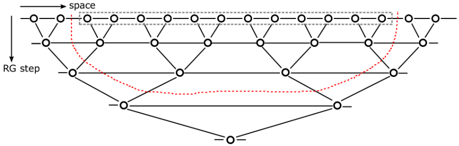

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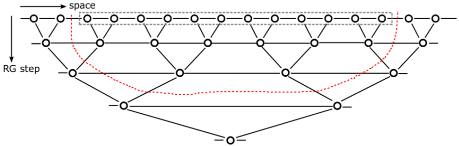

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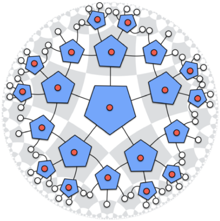

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- **IF** holography indeed could be understood in terms of tensor networks, then this would imply that any system would have a holographic description...

Some words of caution...

- The tensor network constructions do not seem to lead to a description in terms of some bulk fields
- In particular we do not have any kind of bulk action..
- The picture looks nicest on a spatial hypersurface... (but time evolution can be studied...)
- One might worry that a discrete *AdS*-like geometry seems to be built in from the start...

However, the picture is quite appealing and fascinating...

Aim: construct and study an exact tensor network description...

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Why Matrix Product States?

- MPS is an ansatz which does not make any *a-priori* assumptions about the system (like scale invariance); MPS can respect exact translation invariance; it is the simplest kind of tensor network...
- There are some (very) qualitative similarities with holography: Recall the AdS metric

$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$

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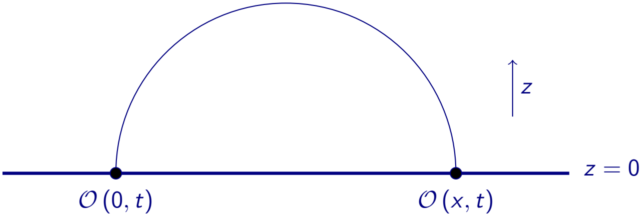
and the picture for computing correlation functions:



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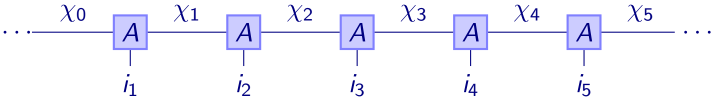

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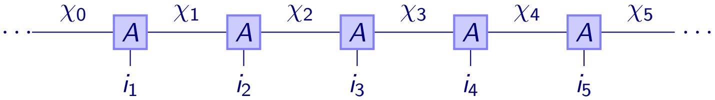

$$\cdots \overset{\chi_0}{—} A_{i_1} \overset{\chi_1}{—} A_{i_2} \overset{\chi_2}{—} A_{i_3} \overset{\chi_3}{—} A_{i_4} \overset{\chi_4}{—} A_{i_5} \overset{\chi_5}{—} \cdots$$

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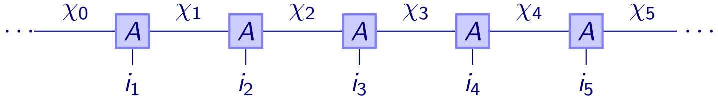

Questions:

- For an *exact* MPS description, the dimension D of the bond space is (generically) $D = \infty$
- **Can this auxiliary infinite dimensional space be interpreted in a natural way as space of modes in an emergent dimension?**
- We would like to compare expressions for holographic and MPS computations of correlation functions...

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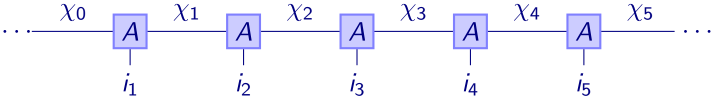

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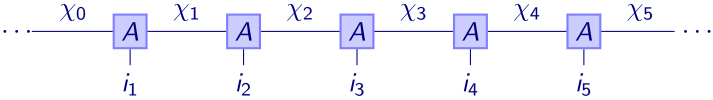

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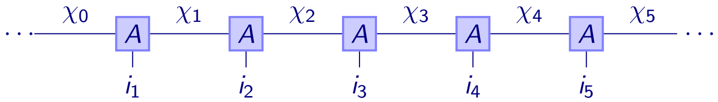

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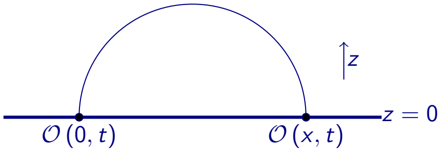

- The holographic prescription for computing the correlation function in a 1+1 dimensional theory is roughly

$$\langle \mathcal{O}(x,t)\, \mathcal{O}(0,t) \rangle = \text{“} \lim_{z \to 0} z^{\#} \langle \Phi(x,t,z) \Phi(0,t,z) \rangle \text{”}$$

- Suppose that the bulk field has a free field expansion

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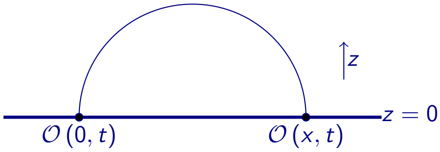
Motivation — understanding holography?



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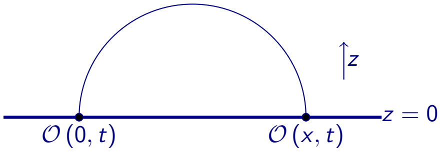

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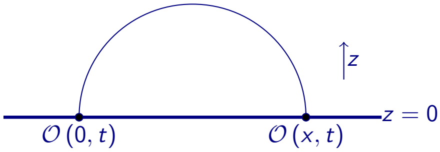

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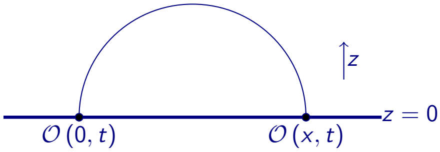

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Motivation — understanding holography?

We would like to evaluate a correlation function using MPS's and see if it can be put naturally in the above form...

Motivation — some questions for integrable QFT's

Another motivation for studying MPS's – not pursued so far...

- An interesting problem in AdS integrability is the interaction of strings...

- Together with Z. Bajnok, we proposed a set of functional equations for the (decompactified) string vertex
- Unfortunately these functional equations have many solutions (like form-factor axioms)
- It would be very interesting to be able to test them for an interacting QFT... (e.g. like sinh-Gordon)
- MPS would be very nice here as they are defined directly in position space...

But this is a completely different (and difficult) project...

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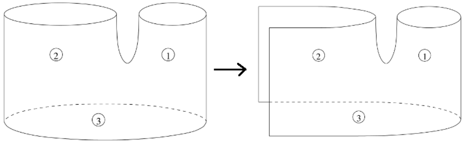

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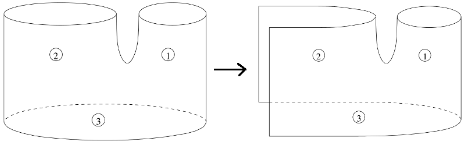

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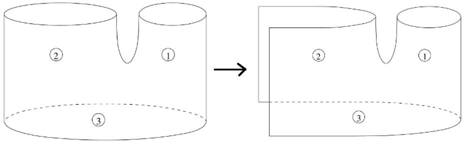

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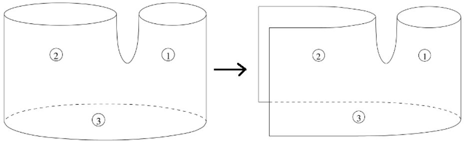

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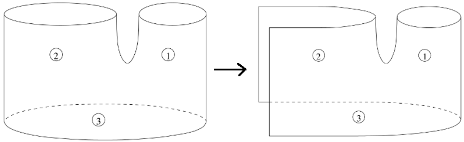

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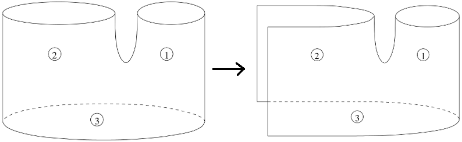

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General construction of exact bosonic MPS

What physical system to choose for the MPS construction?

- We need an *exact* MPS with $D = \infty$ — the system better be exactly solvable...
- We would like to have a system which is not necessarily conformal..
  1. Conformal systems have "too much" symmetry
  2. For conformal systems one is free to make a Weyl transformation of the underlying metric...
- We will pick a discrete lattice system in order to directly apply the original MPS formalism
- ... and not to deal with infinities/renormalization etc.
- But the lattice system will be a discretization of free 1+1 dimensional bosonic QFT

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- Consider the quite generic bosonic lattice system

$$L = \sum_n \frac{M}{2}\dot{\phi}_n^2 - \frac{D}{2}(\phi_{n+1} - \phi_n)^2 - V(\phi_n)$$

- For the quadratic case $V(\phi) = \frac{K}{2}\phi^2$, one can of course write explicitly the wave function in position representation

$$\Psi(\ldots, \phi_{n-1}, \phi_n, \phi_{n+1}, \ldots) = e^{-\frac{1}{2}\sum_{k,l}\phi_k C_{kl}\phi_l}$$

- The above form is unfortunately *very far* from a MPS ansatz as all physical sites ϕ_n are directly coupled to each other...
- We need to *decouple* the physical sites through an intermediate auxillary bond space

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- We can use the standard path integral expression for the ground state wavefunction

$$\Psi(\ldots, \phi_{n-1}, \phi_n, \phi_{n+1}, \ldots) = \lim_{T\to\infty} \int_{\substack{u_n(0)=0 \\ u_n(T)=\phi_n}} \mathcal{D}u_n(\tau) e^{-S_E[\ldots,u_{n-1},u_n,u_{n+1},\ldots]}$$

- Now, we would like to reinterpret the above formula as a product of MPS's.
- **Key question: What are the auxiliary bond variables over which we sum over?**
- First natural guess: use $u_n(\tau)$...
- **Problems:**
  1. Boundary conditions $u_n(T) = \phi_n$ would link physical space and bond variables which should not happen
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General construction of exact bosonic MPS

- We can use the standard path integral expression for the ground state wavefunction

$$\Psi(\ldots, \phi_{n-1}, \phi_n, \phi_{n+1}, \ldots) = \lim_{T\to\infty} \int_{\substack{u_n(0)=0 \\ u_n(T)=\phi_n}} \mathcal{D}u_n(\tau) e^{-S_E[\ldots, u_{n-1}, u_n, u_{n+1}, \ldots]}$$

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$$e^{-\int_0^T \frac{D}{2}(u_{n+1}-u_n)^2 d\tau} = \int \mathcal{D}s_n(\tau)\, \delta(s_n - (u_{n+1} - u_n))\, e^{-\int_0^T \frac{D}{2} s_n^2 d\tau}$$

- Interchange the order of integration and first integrate out $s_n(\tau)$

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- Identify the Lagrange multiplier functions $\chi_n(\tau)$ with the bond variables

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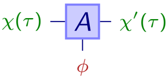

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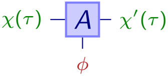

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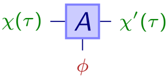

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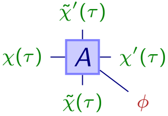

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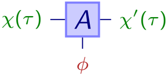

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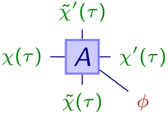

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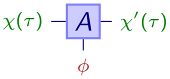

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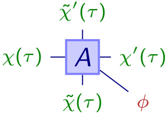

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The Klein-Gordon harmonic chain

- From now on we deal with the quadratic case

$$L = \sum_n \frac{M}{2}\dot{\phi}_n^2 - \frac{D}{2}(\phi_{n+1} - \phi_n)^2 - \frac{K}{2}\phi_n^2$$

- The excitations of the chain have energies

$$E(p) = \sqrt{\frac{K}{M} + \frac{4D}{M}\sin^2\frac{p}{2}} \qquad \text{with } p \in (-\pi, \pi)$$

- The correlation functions are in fact quite complicated

$$\langle \phi_0 \phi_m \rangle = \frac{1}{\sqrt{MK + 2DM}} \cdot \frac{z^m}{2\mu} \binom{m - \frac{1}{2}}{m} {}_2F_1\left(\frac{1}{2}, m + \frac{1}{2}, m + 1; z^2\right)$$

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- The excitations of the chain have energies

$$E(p) = \sqrt{\frac{K}{M} + \frac{4D}{M}\sin^2\frac{p}{2}} \qquad \text{with } p \in (-\pi, \pi)$$

- The correlation functions are in fact quite complicated

$$\langle \phi_0 \phi_m \rangle = \frac{1}{\sqrt{MK + 2DM}} \cdot \frac{z^m}{2\mu} \binom{m - \frac{1}{2}}{m} {}_2F_1\left(\frac{1}{2}, m + \frac{1}{2}, m + 1; z^2\right)$$

where

$$\mu = \frac{1}{\sqrt{1 + z^2}} \qquad z = \frac{1 - \sqrt{1 - \tilde{\alpha}^2}}{\tilde{\alpha}} \qquad \tilde{\alpha} = \frac{2D}{K + 2D}$$

Exact MPS for the Klein-Gordon harmonic chain

- The exact formula for $A^{\phi}[\chi(\tau), \chi'(\tau)]$ is

$$e^{-\frac{M}{2}\phi^2\omega_0 \coth \omega_0 T - i\phi \int_0^T \frac{\sinh \omega_0 \tau}{\sinh \omega_0 T} j(\tau) d\tau - \frac{1}{2M}\int_0^T \int_0^T j(\tau)\Delta(\tau,\tau')j(\tau')d\tau d\tau' - \frac{1}{4D}\int_0^T \chi(\tau)^2 + \chi'(\tau)^2 d\tau}$$

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$$j(\tau) = \chi(\tau) - \chi'(\tau)$$

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Correlation functions using MPS

- We need to define the transfer matrix

- For normalizability, there exists an eigenstate of the transfer matrix with eigenvalue 1

- We can now compute the correlation function

- We need to diagonalize the transfer matrix...

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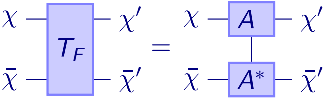

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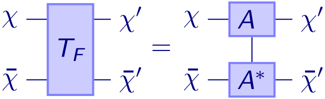

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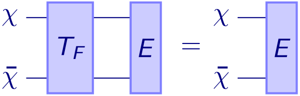

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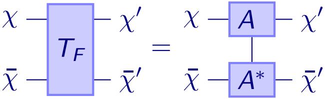

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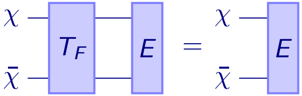

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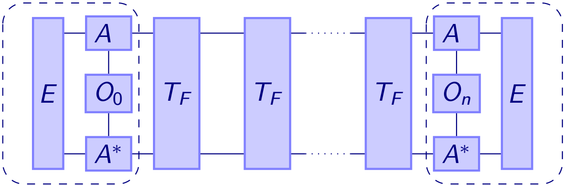

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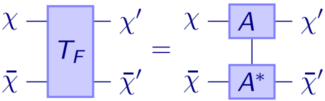

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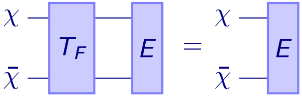

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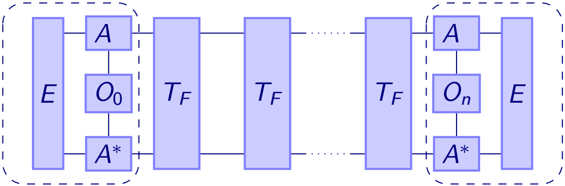

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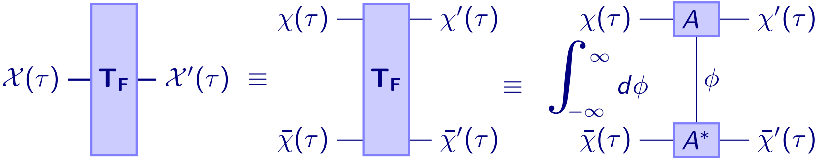

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$$\mathbf{T_F}[\mathcal{X}(\tau),\mathcal{X}'(\tau)] = e^{-\frac{1}{2}\int \mathbf{J}(\tau)\mathbf{\Sigma}(\tau,\tau')\mathbf{J}^T(\tau')d\tau d\tau' - \frac{\alpha}{2}\int \mathcal{X}(\tau)^2 + \mathcal{X}'(\tau)^2 d\tau}$$

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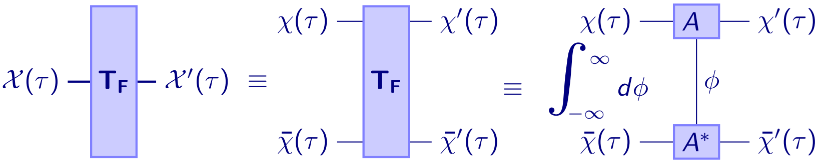

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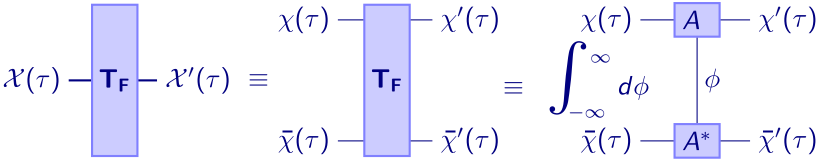

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- Then the *functional* eigenvalue equation decomposes into a product of 1D ones:

$$(normalization) \cdot \int_{-\infty}^{\infty} dx' e^{-\frac{1}{2}(x-x')\Sigma(x-x') - \frac{\alpha}{2}x^2 - \frac{\alpha}{2}x'^2} \psi(x') = \lambda\, \psi(x)$$

- These can be solved in terms of harmonic oscillator wavefunctions:

$$\psi_n(x) \propto H_n(\sqrt{\widetilde{D}}x) e^{-\frac{1}{2}\widetilde{D}x^2} \qquad \lambda_n = \left(\frac{\Sigma}{\alpha + \Sigma + \widetilde{D}} \right)^n$$

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$$E_{T_F}(k) = \log\left(1 + x(k) + \sqrt{x(k)\,(2 + x(k))}\right)$$

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$$E_{T_F}(k) = \log\left(1 + x(k) + \sqrt{x(k)\,(2 + x(k))}\right)$$

with

$$x(k) = \frac{M}{2D}\left(\omega_0^2 + k^2\right)$$

- This is ***exactly*** the **"mirror" dispersion relation** of $AdS_5 \times S^5$ magnons... Ambjorn, RJ, Kristjansen '05

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- "Mirror" dispersion relation describes the dispersion relation of particles appearing in the Thermodynamic Bethe Ansatz for QFT...
- This involves exchanging the role of space and time...
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- We obtained a Fock space of two kinds of modes
- The modes are parametrized by a noncompact momentum $k \in [0, +\infty)$, **distinct** from the physical momenta $p \in (-\pi, \pi)$
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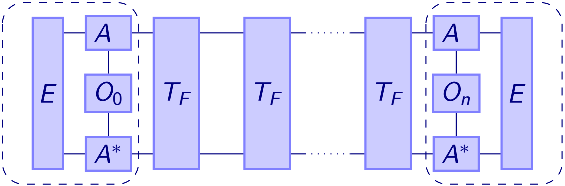
The correlation function $\langle \phi_0 \phi_m \rangle$

Recall the MPS recipe for computing correlation functions:



- It is not difficult to compute the form factors of ϕ_n
- We have a nonvanishing overlap only with (auxiliary) single particle states
- We get thus

$$\langle \phi_0 \phi_m \rangle = \int_0^\infty \frac{dk}{2\pi} f_\phi(k)^2 e^{-(m+1)E_{T_F}(k)}$$

- This can be contrasted with the conventional formula for the correlation function

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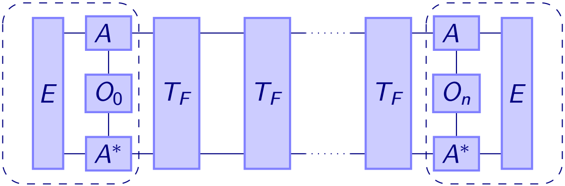

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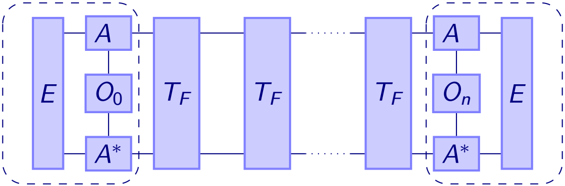

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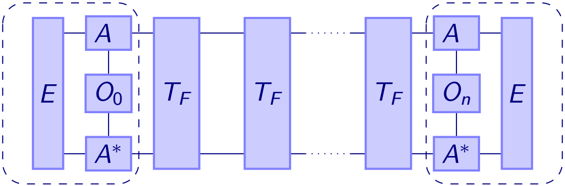

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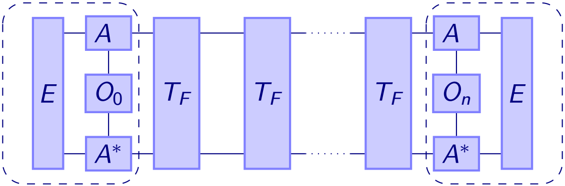

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- In holography, fields in the bulk directly serve to compute correlation functions...
- The potential MPS counterpart of holography should be the MPS contracted with its complex conjugate...
- We have found at *each* lattice site two Fock spaces of modes parametrized by $k \geq 0$

- These modes are in "position space" w.r.t. the lattice. Passing to Fourier modes would give

$$a^\dagger_{k,p},\ a_{k,p} \quad \text{and} \quad b^\dagger_{k,p},\ b_{k,p} \qquad \text{where} \quad k \geq 0,\ p \in [-\pi, \pi]$$

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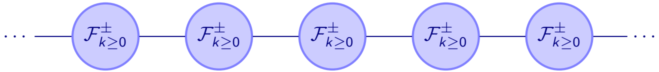

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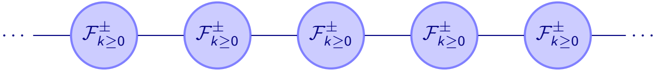

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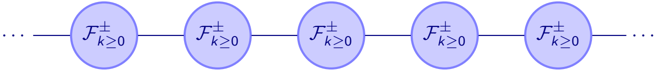

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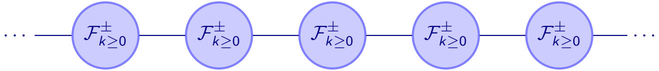

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- On the other hand we have found no trace of an interesting (curved) geometry
- The simple form of $F^2(k, p)$ reflects most probably just the original Euclidean path integral for the wave function
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- We have derived general formulas for MPS or PEPS for interacting bosonic lattice systems
- In principle these formulas could be used as an alternative to conventional variational approach
- We have evaluated explicitly the MPS formula for the bosonic harmonic chain
- Diagonalization of the transfer matrix yields a Fock space of two sets of modes parametrized by $k \in (0, \infty)$ with rather intricate dispersion relations ($\equiv$ mirror dispersion relation)
- Mixed message w.r.t. holographic interpretation:
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